

# Statistical properties of world investment networks

Dong-Ming Song<sup>1,2</sup>, Zhi-Qiang Jiang<sup>1,2</sup>, and Wei-Xing Zhou<sup>1,2,3,4, a</sup>

<sup>1</sup> School of Business, East China University of Science and Technology, Shanghai 200237, China

<sup>2</sup> School of Science, East China University of Science and Technology, Shanghai 200237, China

<sup>3</sup> Research Center for Econophysics, East China University of Science and Technology, Shanghai 200237, China

<sup>4</sup> Research Center of Systems Engineering, East China University of Science and Technology, Shanghai 200237, China

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**Abstract.** We have performed a detailed investigation on the world investment networks constructed from the Coordinated Portfolio Investment Survey (CPIS) data of the International Monetary Fund, ranging from 2001 to 2006. The distributions of degrees and node strengthes are scale-free. The weight distributions can be well modeled by the Weibull distribution. The maximum flow spanning trees of the world investment networks possess two universal allometric scaling relations, independent of time and the investment type. The topological scaling exponent is  $1.17 \pm 0.02$  and the flow scaling exponent is  $1.03 \pm 0.01$ .

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## 1 Introduction

The constituents of a complex system and their interactions can be characterized by a complex network. The network perspective has stimulated explosive interests in the research of social, informational, technological, and biological systems, resulting in deeper understanding of complex systems [1,2,3,4]. As a part of social systems, the network properties of many economic and financial systems have been studied. This literature grows fast and we try to present a very brief review below.

The stocks in a stock market belong to different industrial sectors. Generally speaking, the prices of stocks in the same sector evolve in a correlated manner. If we treat each stock as a node and the distance of two stocks based on the cross-correlation coefficient as the weight linking the two nodes, then the market forms a network. The minimal spanning tree extracted from the distance matrix can be used to investigate the hierarchical structure of a portfolio of stocks, which is usually related to industrial sectors [5, 6,7,8]. Other topological properties of stock market networks are also studied for different markets [9,10,11].

Alternatively, rather than considering a portfolio stocks, the price time series of a single stock can also be mapped into networks, which enables us to investigate the dynamics of a stock through its network structure. There are several methods for this purpose. For a pseudoperiodic time series, one can partition it into disjoint cycles according to the local minima or maxima, and each cycle is considered a basic node of a network, in which two nodes

are deemed connected if the phase space distance or the correlation coefficient between the corresponding cycles is less than a predetermined threshold [12]. We note that a weighted network can also be constructed if the phase space distance or the correlation coefficient is treated as the weigh of a link. This method for pseudoperiodic time series can also be generalized to other time series, which has been applied to stock time series [13]. Other methods for network construction from time series are based on fluctuation patterns [14,15], visibility of nodes [16], and so on.

There are also intense interests in the study of world trade webs, which describe the international trade relations between different economies. Serrano and Boguñá have constructed a world trade web utilizing the COM-TRADE database of the United Nations Statistics Division and pointed out that the world trade web exhibits typical features of complex networks [17]. A fitness model [18] has been proposed to reproduce the topology of world trade webs based on a different and more detailed data set [19], in which the fitness of a node is associated with the GDP of the corresponding economy. The fitness model was then extended to an evolving and directed description of world trade webs [20]. Furthermore, the interrelation between the topology of a world trade web and the GDP of countries has been elaborated [21]. The world trade web can also be described by weighted networks, in which not only the topology but also the trade volume are considered [22,23]. A gravity model was used to model the weighted world trade web [24,23]. The world trade webs also show universal allometric scaling [25], synchroniza-

<sup>a</sup> e-mail: wxzhou@ecust.edu.cn

**Table 1.** The number of nodes  $N_v$ , the number of arcs  $N_a$  (directed), and the number of edges  $N_e$  (undirected) of the world investment networks constructed from different kinds of investment data for different years.

Year	TP			ES			TD			SD			LD		
	$N_v$	$N_a$	$N_e$	$N_v$	$N_a$	$N_e$	$N_v$	$N_a$	$N_e$	$N_v$	$N_a$	$N_e$	$N_v$	$N_a$	$N_e$
2001	195	3244	2397	167	2261	1701	168	2655	1959	120	971	778	168	2649	1961
2002	194	3325	2477	172	2315	1737	174	2710	2042	136	1043	864	172	2716	2044
2003	197	3597	2649	177	2930	2216	176	2910	2193	134	1108	888	170	2893	2167
2004	193	3791	2774	174	3111	2344	173	3136	2339	133	1156	930	165	2968	2258
2005	195	4001	2926	173	2764	2055	170	3349	2482	130	1260	1016	164	3117	2367
2006	210	4483	3248	193	3187	2353	182	3592	2638	145	1353	1080	172	3415	2528

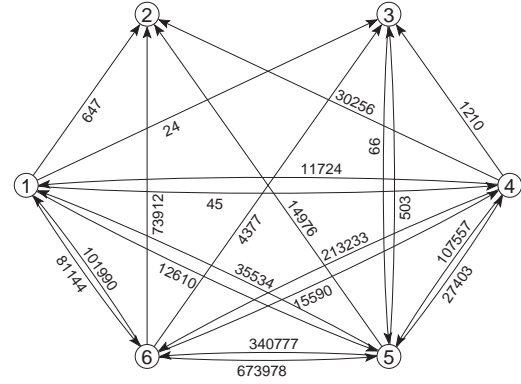
tion [26], community structure [27], and rich-club structure [23].

There are also other economic networks, such as the world exchange arrangements web [28], the “product space” networks [29], the venture capital networks [30,31], the stock investment networks [32], and the bank connection networks [33]. In this work, we shall study the statistical properties of a new economic network named the world investment network (WIN). The remainder of the work is organized as follow. In Section 2, we briefly describe the data sets adopted and construct the world investment networks. We investigate the basic statistical properties of unweighted networks in Section 3 and weighted networks in Section 4. We further study the allometric scaling in Section 5. Section 6 concludes.

## 2 Constructing world investment networks

We construct the world investment networks using the Coordinated Portfolio Investment Survey (CPIS) data released by the International Monetary Fund (IMF). The CPIS data are publicly available and can be retrieved from the IMF web site. There are five kinds of data including total portfolio investment (TP), equity securities (ES), total debt securities (TD), long-term debt securities (LD), and short-term debt securities (SD). All these data are recorded from 2001 to 2006. Therefore, there are 30 tables in total. Each table contains the investment information among different economies.

A directed and weighted network can be constructed from each table. In each network, the nodes represent economies. If economy  $i$  invests in economy  $j$ , we can draw a directed link  $i \rightarrow j$  from node  $i$  to node  $j$ , to which a weight identical to the investment volume is assigned. The directed and weighted network can be fully expressed by a matrix  $W$ , where the element  $w_{ij}$  stands for the investment volume from economy  $i$  to economy  $j$ . We note that  $w_{ii} = 0$  for all economies and  $w_{ij} = 0$  if economy  $i$  does not invest in  $j$ . The matrix  $W$  does not need to be symmetric, that is,  $w_{ij} \neq w_{ji}$ . A schematic diagram is illustrated in Figure 1, which is a part of the network constructed from the ES data in 2001. When  $w_{ij} = 0$ , there is no link from  $i$  to  $j$  in the figure.

**Fig. 1.** Diagram illustrating part of the world investment network obtained from ES in 2001. The numbers 1, 2, 3, 4, 5, and 6 in the open circles signify Australia, China, Egypt, Luxembourg, United Kingdom, and United States, respectively. The numbers above the edges represent the network weights (investment volumes).

The directed network  $W$  can be converted into an undirected network  $G$ , whose weight is determined by

$$g_{ij} = g_{ji} = w_{ij} + w_{ji} . \quad (1)$$

For convenience, directed and undirected links are called arcs and edges, respectively. Let  $N_v$  be the number of nodes of network  $W$ . Then, the number of arcs of  $W$  is

$$N_a = \sum_{i=1}^{N_v} \sum_{j=1}^{N_v} \mathbf{I}(w_{ij}) , \quad (2)$$

where the indicator function  $\mathbf{I}(x)$  equals to 1 if  $x > 0$  and 0 otherwise, and the number of edges of  $G$  is

$$N_e = \frac{1}{2} \sum_{i=1}^{N_v} \sum_{j=1}^{N_v} \mathbf{I}(g_{ij}) = \sum_{i=1}^{N_v} \sum_{j=i}^{N_v} \mathbf{I}(g_{ij}) , \quad (3)$$

It is obvious that  $2N_e \geq N_a$ . Table 1 reports the values of  $N_v$ ,  $N_a$  and  $N_e$  for all the 30 networks. Roughly speaking, for each of the five investment types, the number of nodes

$N_a$  is almost constant before 2006 and increases sharply in 2006, while the number of links ( $N_e$  and  $N_a$ ) increases gradually from 2001 to 2006 for both directed and undirected networks, which implies an increasing globalization.

### 3 Basic statistical properties of unweighted world investment networks

#### 3.1 Undirected networks

An undirected and unweighted network  $A$  can be extracted from an undirected and weighted network  $G$ . We note that  $A$  is the adjacency matrix of  $G$ . The element  $a_{ij}$  of  $A$  can be determined as follows,

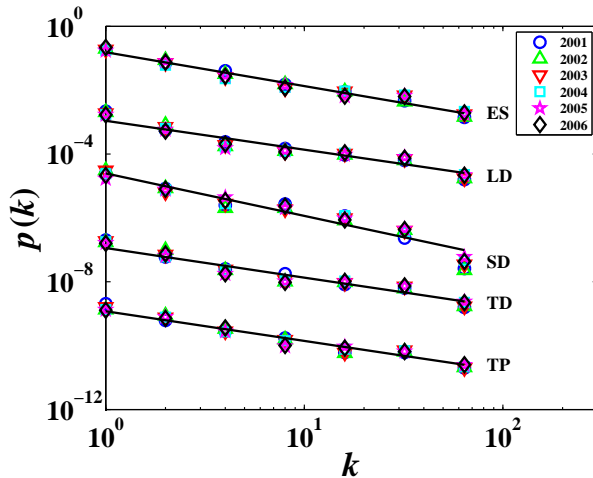
$$a_{ij} = \mathbf{I}(g_{ij}) . \quad (4)$$

Speaking differently,  $a_{ij} = 1$  if economy  $i$  invests in  $j$  or  $j$  invests in  $i$  or both, and  $a_{ij} = 0$  otherwise.

We first investigate the degree distribution of nodes for each network. The degree of node  $i$  can be calculated as follows

$$k_i = \sum_{j=1}^{N_v} a_{ij} . \quad (5)$$

Figure 2 shows the degree distributions for the 30 undirected networks. For clarity, the data for different types of investment are shifted vertically. For the networks constructed from the same investment type, the six degree distributions for different years almost collapse onto a single curve. For the networks from different investment types, the distributions are different from each other.



**Fig. 2.** (Color online) Degree distributions of the constructed undirected WINs. The data points for LD, SD, TD, and TP have been translated vertically by a factor of  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ , and  $10^{-4}$  for clarity. The solid lines are the best power-law fits to the data.

An evident feature of the degree distributions is that they all exhibit a power-law behavior:

$$p(k) \sim k^{-\gamma} . \quad (6)$$

It means that the world investment networks are scale-free. The power-law exponent  $\gamma$  can be estimated through a linear least-squares regression to fit the data in log-log coordinates. Table 2 reports the exponents  $\gamma$  for all the networks. It is found that, for each type of networks, the exponents  $\gamma$  slightly decreases along time. The observation means that there are more highly connected economies recently, also indicating an increase in globalization.

**Table 2.** The power-law exponents  $\gamma$  of the degree distributions for undirected and unweighted networks. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.03(8)	1.12(5)	1.04(9)	1.46(19)	1.04(10)
2002	1.00(8)	1.13(4)	1.04(11)	1.47(20)	1.03(12)
2003	1.00(9)	1.01(10)	1.02(10)	1.39(16)	0.99(12)
2004	0.95(7)	0.95(9)	0.95(10)	1.32(14)	0.92(11)
2005	0.93(9)	1.04(8)	0.91(12)	1.24(12)	0.90(13)
2006	0.93(8)	1.06(10)	0.93(12)	1.33(14)	0.91(11)

The average minimum path length is among the most studied quantities of complex networks. Table 3 lists the path length of all the networks. We see that the values of average minimum path length are quite small. For the TP, TD and LD networks, the average minimum path length decreases. For the ES networks, the average minimum path length does not have a clear trend and reaches a maximum value in 2006. For the SD networks, it is also hard to identify an evident trend. It is noteworthy to point out that the average minimum path length does not change much from one year to another.

**Table 3.** The average path lengths of all the undirected and unweighted networks. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.89	1.91	1.86	1.99	1.86
2002	1.88	1.94	1.90	2.12	1.88
2003	1.87	1.86	1.86	2.03	1.83
2004	1.84	1.84	1.84	2.01	1.81
2005	1.83	1.91	1.80	1.92	1.79
2006	1.84	1.95	1.82	1.97	1.79

The clustering coefficient of a node is a measure of the cluster structure indicating how much the adjacent vertices of its adjacent vertices are its adjacent vertices. In other words, the clustering coefficient of node is the ratio of the number of existing edges between its adjacent vertices to the number of possible edges between them. Table 4 presents the average clustering coefficients for all the networks. The average clustering coefficients of networks with the same investment type are almost the same for different years. A closer scrutiny shows that, the aver-

age clustering coefficient roughly decreases along time for the ES type and increases for the other four types.

**Table 4.** The average clustering coefficients of all the undirected and unweighted networks.

Year	TP	ES	TD	SD	LD
2001	0.67	0.67	0.66	0.52	0.64
2002	0.73	0.66	0.68	0.49	0.66
2003	0.71	0.67	0.67	0.50	0.67
2004	0.73	0.66	0.66	0.55	0.66
2005	0.73	0.67	0.68	0.60	0.67
2006	0.74	0.64	0.70	0.60	0.68

### 3.2 Directed networks

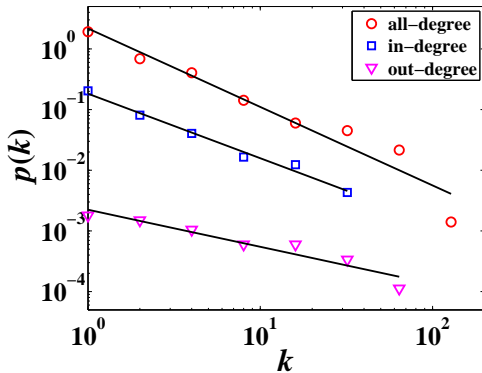
A directed and unweighted network  $B$  is the adjacency matrix of the corresponding directed and weighted network  $W$ . The element  $b_{ij}$  of  $B$  can be determined as follows,

$$b_{ij} = \mathbf{I}(w_{ij}) . \quad (7)$$

Speaking differently,  $b_{ij} = 1$  if economy  $i$  invests in economy  $j$  and  $b_{ij} = 0$  otherwise. The in-degree  $k^{\text{in}}$ , out-degree  $k^{\text{out}}$ , and all-degree  $\gamma^{\text{all}}$  are defined by,

$$k_i^{\text{in}} = \sum_{j=1}^{N_v} b_{ij} , \quad k_i^{\text{out}} = \sum_{j=1}^{N_v} b_{ji} , \quad k_i^{\text{all}} = k_i^{\text{in}} + k_i^{\text{out}} . \quad (8)$$

Figure 3 illustrates the degree distributions of the directed and unweighted network  $B$  constructed from the ES data in 2001.



**Fig. 3.** (Color online) Degree distributions of the directed and unweighted network  $B$  constructed from the ES data in 2001. The data points for in-degree and out-degree have been translated vertically by a factor of 0.1 and 0.01 for clarity. The solid lines are the best power-law fits to the data.

We observe that the probability distributions of the degrees are consistent with a power-law behavior:

$$p(k^{\text{io}}) \sim (k^{\text{io}})^{-\gamma_{\text{io}}} , \quad (9)$$

where io = in for in-degrees, io = out for out-degrees, and io = all for all-degrees. Linear least-squares regressions give the estimates of the three power-law exponents:  $\gamma^{\text{all}} = 1.17 \pm 0.07$  for all-degrees,  $\gamma^{\text{in}} = 1.06 \pm 0.07$  for in-degrees, and  $0.58 \pm 0.06$  for out-degrees, respectively. The exponents  $\gamma^{\text{all}}$ ,  $\gamma^{\text{in}}$ , and  $\gamma^{\text{out}}$  for the all the directed and unweighted networks are reported in Table 5. On average, the exponents  $\gamma^{\text{all}}$ ,  $\gamma^{\text{in}}$ , and  $\gamma^{\text{out}}$  decrease along time for each investment type. This shows that the investments among economies become much denser from year to year, also a signal of an increasing globalization from 2001 to 2006.

**Table 5.** The power-law exponents  $\gamma^{\text{all}}$  (top panel),  $\gamma^{\text{in}}$  (middle panel) and  $\gamma^{\text{out}}$  (bottom panel) of the degree distributions for all the directed and unweighted networks. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.05(6)	1.17(7)	1.00(7)	1.20(14)	1.00(7)
2002	1.02(5)	1.12(5)	1.03(7)	1.19(10)	0.99(8)
2003	1.01(5)	1.00(6)	1.00(6)	1.25(12)	0.97(8)
2004	0.97(3)	0.98(6)	0.97(7)	1.15(6)	0.94(8)
2005	0.97(7)	1.04(6)	0.94(8)	1.12(7)	0.94(9)
2006	0.97(6)	1.02(7)	0.96(8)	1.22(5)	0.94(7)
2001	1.03(8)	1.06(6)	0.95(8)	1.38(7)	0.95(9)
2002	0.83(6)	1.04(7)	1.50(31)	1.42(7)	0.85(14)
2003	0.89(7)	1.01(9)	1.05(10)	1.37(10)	0.91(11)
2004	0.77(7)	0.91(8)	0.88(8)	1.30(7)	0.83(11)
2005	0.77(11)	1.08(7)	0.79(13)	1.20(7)	0.78(13)
2006	0.73(12)	1.03(8)	0.75(13)	1.29(5)	0.71(12)
2001	0.35(15)	0.58(7)	0.16(15)	0.81(17)	0.17(16)
2002	0.42(16)	0.51(10)	0.32(11)	0.75(18)	0.31(16)
2003	0.53(24)	0.19(14)	0.19(15)	0.85(15)	0.18(14)
2004	0.23(17)	0.18(19)	0.17(19)	0.75(11)	0.19(20)
2005	0.31(18)	0.15(15)	0.19(16)	0.91(13)	0.24(16)
2006	0.05(10)	0.23(8)	0.16(18)	0.64(27)	0.14(27)

**Table 6.** The average minimum path lengths of all the directed and unweighted networks.

Year	TP	ES	TD	SD	LD
2001	4.68	4.24	4.16	3.82	4.16
2002	4.65	4.48	4.32	4.58	4.30
2003	4.50	4.28	4.21	4.26	4.09
2004	4.28	4.01	3.94	4.05	3.85
2005	4.24	4.09	3.72	3.76	3.72
2006	4.51	4.46	4.05	4.13	3.87

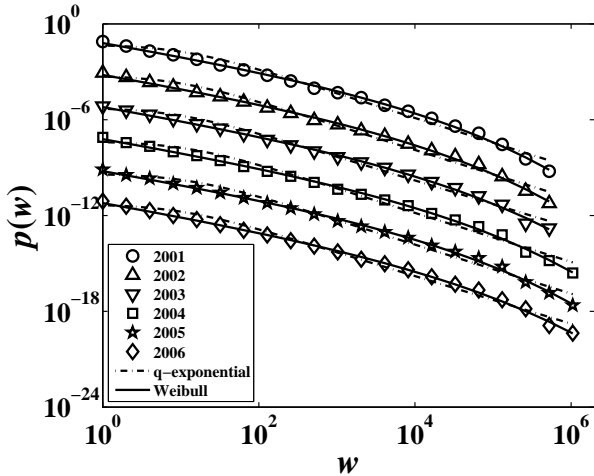
We also report in Table 6 the average minimum path lengths of the constructed directed networks. We find that, for each type of investment, the average minimum path length gradually decreases from 2001 to 2005, followed by a sharp increase in 2006. This is not surprising and can be explained as follows. According to Table 1, the number of nodes or economies increases abruptly in

2006, which have few links connecting to other nodes. This considerably increases the average minimum path length of a directed network. The increase of the average minimum path length in 2006 does not mean a weakening globalization in 2006. On the contrary, the inclusion of more economies in the networks indicates a speedup in the globalization.

## 4 Basic statistical properties of weighted world investment networks

### 4.1 Distribution of arc weights

Figure 4 plots the empirical probability distributions of weights for the six directed and weighted networks constructed using the TP data from 2001 to 2006. No clear evidence of power laws is observed. We find that the results are very similar for the ES, TD, SD and LD data (see Figure A1 in the Appendix).



**Fig. 4.** Empirical probability density functions of weights of the directed and weighted networks constructed using the TP data from 2001 to 2006. The curves have been shifted for clarity. The solid and dot-dashed lines are the best fits with the Weibull and  $q$ -exponential distributions.

We apply the Weibull and the  $q$ -exponential distributions to model the weight distributions [34,35]. The Weibull probability density  $p_w(w)$  can be written as

$$p_w(w) = \alpha\beta w^{\beta-1} \exp(-\alpha w^\beta), \quad (10)$$

and its complementary (cumulative) distribution function  $C_w(w)$  is

$$C_w(w) = \exp(-\alpha w^\beta). \quad (11)$$

When  $\beta = 1$ ,  $p_w(w)$  recovers the exponential distribution. When  $0 < \beta < 1$ ,  $p_w(w)$  is a stretched exponential or sub-exponential. When  $\beta > 1$ ,  $p_w(w)$  is a super-exponential. The  $q$ -exponential probability density  $p_q(w)$  is defined by

$$p_q(w) = \mu [1 + (1 - q)(-\mu w)]^{\frac{1}{1-q}}, \quad (12)$$

and its complementary cumulative distribution function  $C_q(w)$  is

$$C_q(w) = [1 + (1 - q)(-\mu w)]^{\frac{1}{1-q}}. \quad (13)$$

Usually, we have  $q > 1$ . When  $(1 - q)(-\mu w) \gg 1$ , we observe a power-law behavior in the tail  $C_q(w) \sim w^{-1/(q-1)}$  with a tail exponent of  $1/(q - 1)$ .

We adopt the nonlinear least-squares estimator (NLSE) to calibrate the weight distributions. The objective function in the fitting is  $\sum [\ln p(w) - \ln \hat{p}(w)]^2$  rather than  $\sum [p(w) - \hat{p}(w)]^2$ , where  $\hat{p}(w)$  is the empirical data. The resultant fits are also illustrated in Figure 4. The parameters of the two models and the corresponding root-mean-square values  $\chi_w$  and  $\chi_q$  of the fitting residuals are shown in Table 7. It is evident from Figure 4 that the Weibull is a better model than the  $q$ -exponential, which is confirmed by the much smaller values of  $\chi_q$  compared with  $\chi_w$  in Table 7. We also find that the values of  $\alpha$  and  $\beta$  remain constant from 2001 to 2006. Very similar results are observed for the ES, TD, SD and LD networks (see Table A1 and Table A2 in the Appendix).

**Table 7.** Estimated parameters ( $\alpha$ ,  $\beta$ ,  $\mu$  and  $q$ ) and the RMS of fitting residuals ( $\chi_w$  and  $\chi_q$ ).

Year	Weibull			$q$ -exponential		
	$\alpha$	$\beta$	$\chi_w$	$\mu$	$q$	$\chi_q$
2001	0.44	0.22	0.0010	0.05	2.87	0.0024
2002	0.42	0.23	0.0006	0.06	2.89	0.0014
2003	0.43	0.22	0.0001	0.06	3.07	0.0007
2004	0.42	0.22	0.0003	0.04	2.87	0.0018
2005	0.40	0.22	0.0003	0.04	2.86	0.0018
2006	0.43	0.21	0.0004	0.05	3.03	0.0014

### 4.2 Distribution of node strength

For weighted networks, the node strength is a generalization of the degree, which is defined by

$$s_i = \sum_{j=1}^{N_v} w_{ij} + w_{ji}. \quad (14)$$

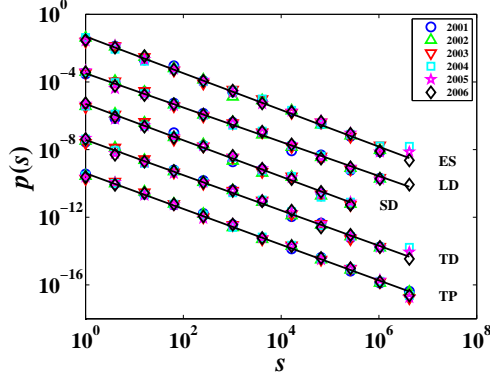
The node strength distributions of all the directed and weighted networks are illustrated in Figure 5.

For all the node strength distributions shown in Figure 5), we see nice power-law behaviors

$$p(s) \sim s^{-\gamma_s} \quad (15)$$

The solid lines are the best fits to the data. The power-law exponents  $\gamma_s$  are depicted in Table 8. These power-law exponents are close to each other for each investment type. For the TP networks, the exponents exhibit a clear decreasing trend from 2001 to 2006.





**Fig. 5.** (Color online) Node strength distributions of the directed and weighted networks. The curves have been shifted vertically for clarity.

**Table 8.** The power-law exponents  $\gamma_s$  of the node strength distributions. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.13(4)	1.12(3)	1.07(2)	1.11(4)	1.08(3)
2002	1.13(4)	1.13(7)	1.08(2)	1.13(4)	1.08(2)
2003	1.11(4)	1.06(2)	1.06(2)	1.11(4)	1.06(2)
2004	1.09(3)	1.11(4)	1.11(4)	1.10(3)	1.04(2)
2005	1.08(3)	1.13(4)	1.10(4)	1.07(3)	1.02(2)
2006	1.07(2)	1.13(4)	1.10(4)	1.08(2)	1.09(4)

## 5 Universal allometric scaling laws

### 5.1 Unweighted world investment networks

Allometric scaling laws are ubiquitous in networking systems such as metabolism of organisms and ecosystems river networks, food webs, and so on [36, 37, 38, 39, 40, 41, 42]. For economic systems, the world trade webs also exhibits a universal allometric scaling [25]. For a complex network, a minimal spanning tree can be extracted. Each node of the tree is assigned a number 1. Two values  $A_i$  and  $C_i$  are defined for each node  $i$  in a recursive manner as follows:

$$A_i = \sum_{j \in \mathbf{J}(i)} A_j + 1, \quad (16)$$

and

$$C_i = \sum_{j \in \mathbf{J}(i)} C_j + A_i, \quad (17)$$

where  $\mathbf{J}(i)$  stands for the set of daughter nodes of  $i$  and  $A_1 = C_1 = 1$  [40]. The allometric scaling relation is then highlighted by the power law relation between  $C_i$  and  $A_i$ :

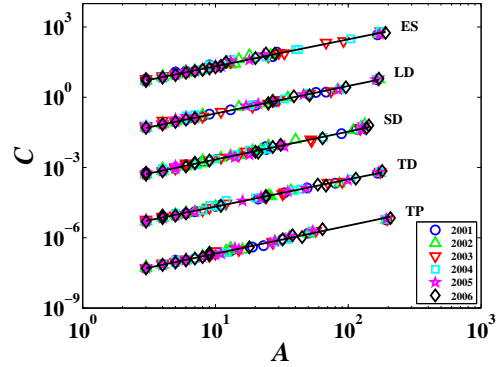
$$C \sim A^\eta. \quad (18)$$

We note that not all trees have such allometric scaling, such as the classic Cayley trees [43].

For spanning trees extracted from transportation networks, the power law exponent  $\eta$  is a measure of transportation efficiency [40, 42]. The smaller is the value of  $\eta$ ,

the more efficient is the transportation. Any spanning tree can range in principle between two extremes, that is, the chain-like trees and the star-like trees. A chain tree has one root and one leaf with no branching. For chain-like trees, it is easy to show that  $A_i = i$  and  $C_i = i(i+1)/2$ . Asymptotically, we have a power between  $C_i$  and  $A_i$  with the exponent  $\eta = 2^-$ . For star-like trees of size  $n$ , there are one root and  $n-1$  leaves directly connected to the root. We have  $A = C = 1$  for all the leaves and  $A = n$  and  $C = 2n-1$  for the root. It follows approximately that  $\eta = 1^+$ . Therefore, if the relation (18) holds, the exponent should be  $1 < \eta < 2$ .

For each undirected and unweighted network, a minimal spanning tree can be obtained. The calculated  $C$  is plotted in Fig. 6 as a function of  $A$  for each network. Nice power-law relations are observed between  $C$  and  $A$ . The points ( $A = 1, C = 1$ ) for the leaves are not shown [42]. For each investment type, the data points of the six networks collapse onto a single curve.



**Fig. 6.** (Color online) Allometric scaling relationship between  $S$  and  $A$ . The data points for LD, SD, TD, and TP are translated vertically for clarity. The solid lines are the best power-law fits to the data.

Nice power-law behaviors are observed in Figure 6. A linear fit of  $\ln C$  against  $\ln A$  gives the estimate of  $\eta$  for each network. The trivial point ( $A = 1, C = 1$ ) should be excluded from the fitting [42]. The resulting exponents are listed in Table 9. We find that the exponents are almost the same for different investment types and different years, which means that the power-law allometric scaling is universal for the world investment networks. This value of  $\eta$  is comparable to  $\eta = 1.13 \sim 1.16$  for food webs [42], but much smaller than  $\eta = 1.3$  for world trade webs [25] and  $\eta = 1.5$  for river networks [40].

### 5.2 Weighted world investment networks

When studying the world trade webs, Duan proposed a framework of flow allometric scaling analysis for weighted networks [25],

$$fA_i = \sum_{j \in \mathbf{J}(i)} fA_j + w_i, \quad (19)$$

**Table 9.** The topological scaling exponents  $\eta$  for all the undirected and unweighted networks. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.15(3)	1.10(4)	1.16(1)	1.19(2)	1.16(2)
2002	1.14(3)	1.15(4)	1.16(2)	1.27(3)	1.15(2)
2003	1.18(3)	1.16(2)	1.16(2)	1.20(3)	1.13(3)
2004	1.18(3)	1.17(3)	1.17(3)	1.21(2)	1.14(3)
2005	1.18(3)	1.14(3)	1.17(2)	1.18(3)	1.13(3)
2006	1.19(3)	1.15(3)	1.16(2)	1.19(2)	1.18(2)
mean	1.17(2)	1.15(2)	1.16(1)	1.21(3)	1.15(2)

and

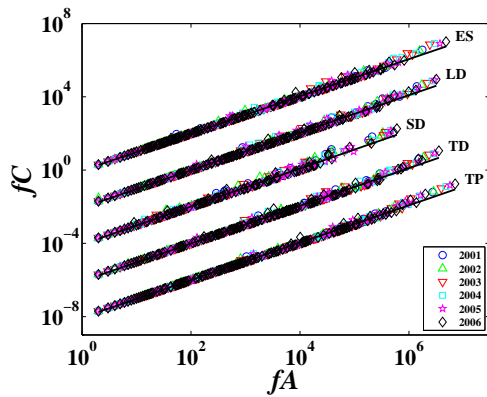
$$fC_i = \sum_{j \in \mathbf{J}(i)} fC_j + fA_i, \quad (20)$$

where  $\mathbf{J}(i)$  is the set of daughter nodes of  $i$ ,  $w_i$  stands for the total weight flowed into node  $i$ ,  $fA_i$  is the weighted quantity of resources and  $fC_i$  stands for the weighted transferring cost. For the leaves, the carried values of  $fA$  and  $fC$  are identical to the investment volumes. One can also expect for certain network that the allometric scaling relation exists between  $fC_i$  and  $fA_i$ :

$$fC \sim fA^\zeta, \quad (21)$$

in which the exponent  $\zeta$  is called the flow allometric scaling exponent [25].

We adopt this analysis on the maximum-flow spanning trees of the undirected and weighted world investment networks. The maximum-flow spanning trees of the investment networks can be obtained [25]. Figure 7 plots  $fC$  with respect to  $fA$  in double logarithmic coordinates. For each investment type, the data points of the six networks collapse onto a single curve, independent of the time.

**Fig. 7.** (Color online) Allometric scaling relationship between  $fC$  and  $fA$ . The data points for LD, SD, TD, and TP are translated vertically for clarity. The solid lines are the best power-law fits to the data.

Evident power-law behaviors are observed between  $fC$  and  $fA$  for all the weighted networks under investigation.

The curves seem parallel for different types of networks. The flow scaling exponents  $\zeta$  are estimated by the slopes of the linear fits of  $\ln fC$  with respect to  $\ln fA$ . The exponents  $\zeta$  are reported in Table 10. We find that the flow allometric scaling exponents are almost the same for different investment types and different years, which means that the power-law allometric scaling is universal for the world investment networks. It is interesting to point out that this  $\zeta$  value of the world investment networks is close to the flow allometric scaling exponent of the world trade webs [25].

**Table 10.** The flow scaling exponents  $\zeta$  for all the undirected and weighted networks. The numbers in the parentheses are the errors magnified by 100.

Year	TP	ES	TD	SD	LD
2001	1.02(0)	1.01(0)	1.03(0)	1.07(1)	1.03(0)
2002	1.02(0)	1.01(0)	1.02(0)	1.06(1)	1.02(0)
2003	1.02(0)	1.03(0)	1.03(1)	1.05(1)	1.02(0)
2004	1.02(0)	1.03(0)	1.03(0)	1.04(1)	1.03(1)
2005	1.02(0)	1.01(0)	1.03(0)	1.04(1)	1.03(1)
2006	1.02(0)	1.01(0)	1.03(0)	1.04(1)	1.02(0)
mean	1.02(0)	1.02(1)	1.03(0)	1.05(1)	1.03(1)

## 6 Conclusion

In this work, we have constructed a new type of economic networks based on the Coordinated Portfolio Investment Survey data released by the International Monetary Fund, which records data from 2001 to 2006. We have studied the statistical properties of these world investment networks. Our results show that there is an increasing globalization in the past few years under investigation.

The degree distributions are scale-free for all the constructed networks. For the same investment type of data, the average path length and average clustering coefficient are almost the same for different years with a weak trend. When we regard the world investment networks as weighted networks, the Weibull and  $q$ -exponential distributions are utilized to fit the weight distribution by means of a non-linear least-squares estimator. We find that the Weibull model remarkably outperforms the  $q$ -exponential model. In addition, the node strength distributions are found to exhibit nice power-law behaviors.

We also investigated the allometric scaling of the minimal spanning trees and the maximum-flow spanning trees of the world investment networks. There are two universal allometric scaling exponents characterizing the topological structure and the investment pattern of the networks. We find that the topological scaling exponent is  $\eta = 1.17 \pm 0.02$  and the flow scaling exponent is  $\zeta = 1.03 \pm 0.01$ . The topological scaling exponent is found to be close to that of the food webs and smaller than that of the world trade webs, while the flow scaling exponent is comparable to that of the world trade webs.

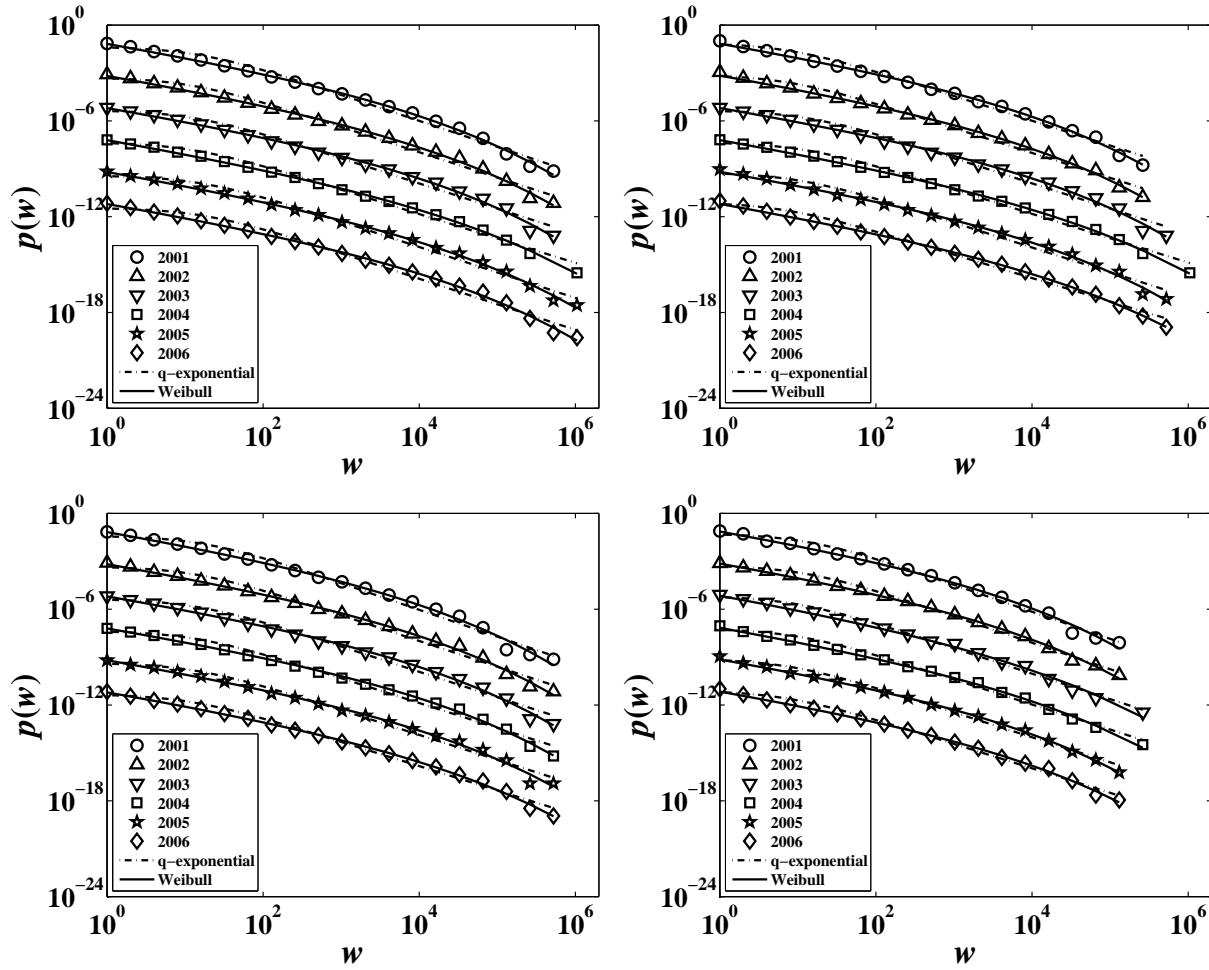
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**Fig. A1.** Empirical probability density function of weights in the constructed networks for different kinds of investments and different years. The solid and dash-dotted lines are the maximum likelihood fits to the Weibull and  $q$ -exponential distributions, respectively.

**Table A1.** Estimated values of parameters  $(\alpha, \beta)$  by means of NLSE. The values of  $\chi_w$  have been multiplied by 100.

Year	TP			ES			TD			SD			LD		
	$\alpha$	$\beta$	$\chi_w$	$\alpha$	$\beta$	$\chi_w$	$\alpha$	$\beta$	$\chi_w$	$\alpha$	$\beta$	$\chi_w$	$\alpha$	$\beta$	$\chi_w$
2001	0.44	0.22	0.10	0.46	0.23	0.13	0.40	0.23	0	0.38	0.24	0.01	0.46	0.22	0.17
2002	0.42	0.23	0.06	0.46	0.22	0.17	0.42	0.23	0.04	0.42	0.23	0.03	0.42	0.23	0.04
2003	0.43	0.22	0.01	0.38	0.24	0.01	0.39	0.23	0.01	0.41	0.23	0.02	0.39	0.23	0.01
2004	0.42	0.22	0.03	0.40	0.23	0	0.40	0.23	0	0.41	0.23	0.02	0.40	0.23	0.01
2005	0.40	0.22	0.03	0.44	0.22	0.10	0.42	0.22	0	0.44	0.22	0.04	0.43	0.22	0
2006	0.43	0.21	0.04	0.42	0.22	0.12	0.40	0.22	0.01	0.44	0.22	0.04	0.42	0.22	0.01

**Table A2.** Estimated values of parameters  $(\mu, q)$  by means of NLSE. The values of  $\chi_w$  have been multiplied by 100.

Year	TP			ES			TD			SD			LD		
	$\mu$	$q$	$\chi_q$	$\mu$	$q$	$\chi_q$	$\mu$	$q$	$\chi_q$	$\mu$	$q$	$\chi_q$	$\mu$	$q$	$\chi_q$
2001	0.05	2.87	0.24	0.07	2.80	0.22	0.04	2.74	0.11	0.05	2.91	0.07	0.06	2.84	0.30
2002	0.06	2.89	0.14	0.06	2.84	0.30	0.06	2.88	0.14	0.05	2.77	0.17	0.05	2.82	0.15
2003	0.06	3.07	0.07	0.05	2.91	0.07	0.05	2.91	0.07	0.05	2.86	0.10	0.05	2.87	0.07
2004	0.04	2.87	0.18	0.04	2.74	0.11	0.04	2.73	0.10	0.05	3.03	0.10	0.05	3.08	0.06
2005	0.04	2.86	0.18	0.05	2.88	0.24	0.04	2.84	0.10	0.06	3.09	0.13	0.05	3.08	0.07
2006	0.05	3.03	0.14	0.05	2.97	0.25	0.04	2.84	0.13	0.06	3.17	0.13	0.05	3.18	0.08